International Journal Of Innovative Trends In Multi Disciplinary Scientific Research
ISSN (Online): 3139 – 2687
(www.ijtmsr.co.in) Volume 9, Issue 6, 2026

Structural and Modal Analysis of a Spur Gear and Pinion Assembly Using Analytical and Numerical Methods

Kandrakota Kranthi kumar[1], K. V. Ganesh[2]

[1]PG scholar, Department of Mechanical engineering, Gonna Institute of Information Technology & Sciences, Visakhapatnam, India.
[2]Assistant professor, Department of Mechanical engineering, Gonna Institute of Information Technology & Sciences, Visakhapatnam, India.

Abstract

Gears are critical mechanical components used for efficient power transmission in industrial, automotive, and aerospace applications. This work presents a comprehensive structural and modal analysis of a spur gear and pinion assembly using theoretical calculations, computer-aided design, and finite element analysis (FEA). Three-dimensional models of the gear and pinion were developed in CATIA and analyzed in ANSYS Workbench, while MATLAB was used to validate the FEA results analytically. Static structural analysis was performed to evaluate stress distribution and deformation under four operating conditions (3500–5000 RPM, corresponding torques of 22,509.55–32,156.51 N·m), and modal analysis was carried out to determine the first three natural frequencies and mode shapes of both components. The maximum stress in the gear reduced from 51.35 MPa to 35.94 MPa and in the pinion from 88.05 MPa to 61.64 MPa as speed increased from 3500 to 5000 RPM, with deformation trends following the same pattern. Comparison between MATLAB and ANSYS results showed excellent agreement (within 0.4%–3.2% for stress and deformation, and within 1.1% for higher-order natural frequencies), confirming the accuracy of the developed models. All obtained natural frequencies were found to be significantly higher than the operating excitation frequencies, indicating a negligible risk of resonance. The results confirm that the AISI 4340 spur gear and pinion assembly possesses adequate strength, stiffness, and dynamic stability for reliable operation in practical engineering applications.

Keywords: Spur gear; pinion; finite element analysis; modal analysis; natural frequency; contact stress.

I. INTRODUCTION

Gears are indispensable machine elements used for power transmission across a wide range of sectors, including automotive, aerospace, energy, manufacturing, robotics, and medical equipment. Among the various gear types—spur, helical, bevel, worm, planetary, and rack-and-pinion—spur gears remain the most widely used owing to their simplicity, high efficiency, low cost, and precise motion transfer between parallel shafts. The reliability of a gear transmission system is governed by its ability to withstand cyclic mechanical loading while maintaining structural integrity and avoiding resonance-induced failure. Consequently, accurate prediction of stress distribution, deformation, and dynamic characteristics such as natural frequency and mode shape is essential during the design stage.

Finite Element Analysis (FEA) has become the standard numerical tool for evaluating gear strength and vibration behaviour, since closed-form analytical methods alone cannot capture complex geometry, contact conditions, and load distribution with sufficient accuracy. This paper combines analytical (MATLAB-based) and numerical (ANSYS-based) approaches to cross-validate the structural and modal response of a spur gear and pinion pair designed for an 11.8 kW motor, operating across a range of speeds and torques.

II. LITERATURE REVIEW

A substantial body of research has addressed the strength, contact stress, mesh stiffness, and vibration behaviour of spur

International Journal Of Innovative Trends In Multi Disciplinary Scientific Research
ISSN (Online): 3139 – 2687
(www.ijtmsr.co.in) Volume 9, Issue 6, 2026

gears. Tian et al. studied dynamic modeling and stability of spur gear systems considering bearing clearance and backlash, while Wang et al. investigated loading capacities of high-contact-ratio internal spur gearing. Zhao et al. developed a fractal contact model incorporating surface roughness effects on normal contact stiffness. Karpat et al. experimentally measured single-tooth stiffness and validated results using ANSYS Workbench, reporting close agreement between numerical and experimental data. Seok-Chul Hwang et al. compared contact stress variation during gear rotation against AGMA standards using FEA, and Putti Srinivasa Rao et al. evaluated contact stresses for aluminium, grey cast iron, and structural steel gears using ANSYS and Hertzian equations.

Further studies by Miryam B. Sánchez et al., Xiang Dai et al., and Shuting Li et al. examined bending and contact strength using combined analytical–FEA methods, while Xihui Liang et al. and Chen et al. analyzed the effect of tooth pitting and crack propagation on mesh stiffness. Investigations by Yılmaz et al. and Sanjay K. Khavdu et al. extended the scope to hybrid and rim-thickness-modified gears, examining the effect on natural frequency and dynamic behaviour. Collectively, the literature establishes that FEA results, when validated against analytical or experimental data, provide reliable predictions of gear strength and vibration response—a methodology adopted in the present study for a spur gear and pinion assembly manufactured from AISI 4340 steel.

III. METHODOLOGY

A. Aim and Objectives

The aim of this work is to analyze the strength of a spur gear and pinion assembly, and to determine its resonance frequencies, using finite element and analytical (mathematical) methods. The specific objectives are: (i) development of three-dimensional CAD models of the gear and pinion; (ii) determination of stress distribution within the gear tooth and body through FEA; (iii) prediction of natural frequencies and mode shapes through modal analysis; and (iv) development of a MATLAB-based analytical model to validate the FEA results.

B. Design Data and Material

The gear and pinion were sized for an 11.8 kW motor operating at four rotational speeds (3500, 4000, 4500 and 5000 RPM), corresponding to torques of 32,156.51, 28,136.94, 25,010.62 and 22,509.55 N·m respectively, computed from $P = 2\pi n_1 M_t/60$. The pinion has 18 teeth and a pitch diameter of 36 mm, while the gear has 72 teeth and a pitch diameter of 144 mm, giving a gear ratio of 4:1, module 2 mm and a 20° pressure angle. Both components are manufactured from AISI 4340 chromium–molybdenum low-alloy steel (Young's modulus E = 200 GPa, Poisson's ratio $\nu = 0.3$, density $\rho = 7850$ kg/m³), selected for its high strength, toughness, machinability and oxidation resistance.

C. Analytical Formulation

Standard gear design relations from Maitra's Handbook of Gear Design and the PSG Design Data Book were used to compute pitch, base, tip and root circle diameters, tooth thickness, centre distance, and face width. The induced bending stress was evaluated using the Lewis-form relation

$$\sigma_b = [(i+1)/(a \cdot m \cdot b \cdot y)] \cdot M_t \quad (1)$$

where i is the gear ratio, a the centre distance, m the module, b the face width, y the Lewis form factor and M_t the pinion torque. The induced contact stress was computed from the Hertzian relation

$$\sigma_c = 0.74[(i+1)/a] \cdot \sqrt{\{[(i+1)/(i \cdot b)] \cdot E \cdot [M_t]\}} \quad (2)$$

with E the equivalent Young's modulus of the mating pair. Tooth deflection was modeled as a cantilever beam, $\delta = W \cdot l^3/(3 \cdot E \cdot I)$, where W is the tangential tooth load, l the tooth length and I the second moment of area of the tooth cross-section. Natural frequencies of the gear and pinion teeth, idealized as cantilever beams, were obtained from

$$f_n = (\lambda_n^2/2\pi l^2) \cdot \sqrt{(EI/\rho A_r)} \quad (3)$$

using the first three Euler–Bernoulli frequency coefficients λ_n = 1.875, 4.694 and 7.855, cross-sectional area A_r, and tooth root moment of inertia I. All formulations were coded in MATLAB

International Journal Of Innovative Trends In Multi Disciplinary Scientific Research
ISSN (Online): 3139 – 2687
(www.ijtmsr.co.in) Volume 9, Issue 6, 2026

(see Appendix, in the source thesis) to compute stress, deformation and natural frequency for each of the four load cases.

D. CAD Modeling and FE Setup

Three-dimensional solid models of the spur gear and pinion were generated in CATIA V5 using the Part Design and Assembly modules, based on the involute tooth geometry derived from the standard gear equations, and exported in IGES format. The assembly was imported into ANSYS Workbench, where AISI 4340 material properties were assigned and bonded contact was defined between each gear/pinion and its shaft. Both shafts were constrained with frictionless support, and the design torques were applied as moment loads about the shaft axis. The assembly was discretized using a hexahedral-dominant solid mesh (SOLID 200-series elements), producing a finite element model with 19,853 elements and 90,632 nodes. Static structural analysis was performed to obtain equivalent (von-Mises) stress and total deformation, followed by a linear modal analysis (Block Lanczos extraction) to obtain the first three natural frequencies and mode shapes of the gear and pinion.

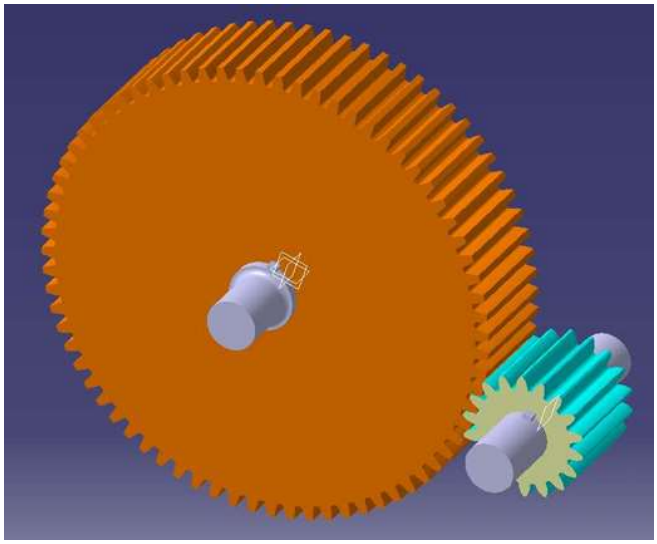


Fig 1: Gear and pinion meshing.

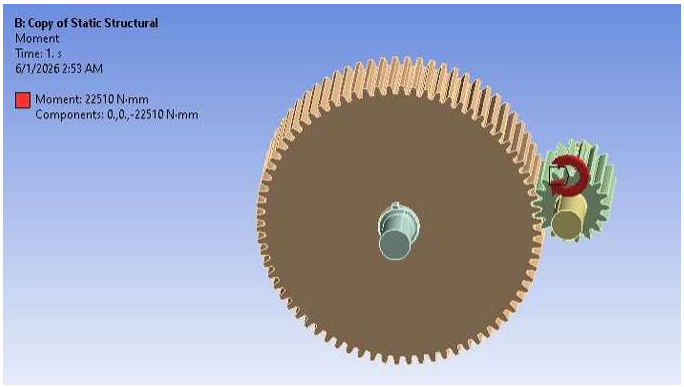


Fig 2: Loading applied to gear and pinion assembly.

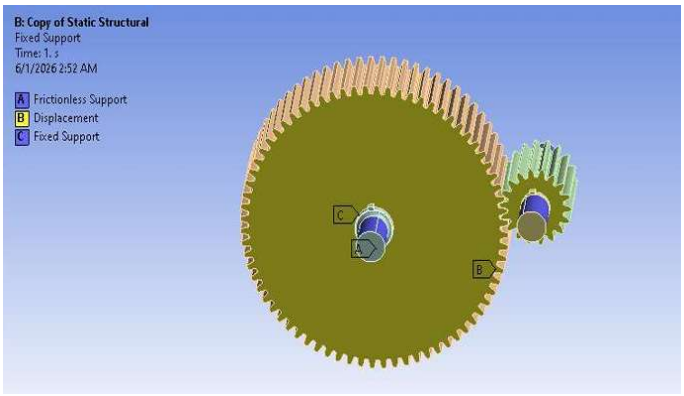


Fig 3: supports and boundary condition of gear and pinion.

IV. RESULTS AND DISCUSSION

A. Static Structural Results

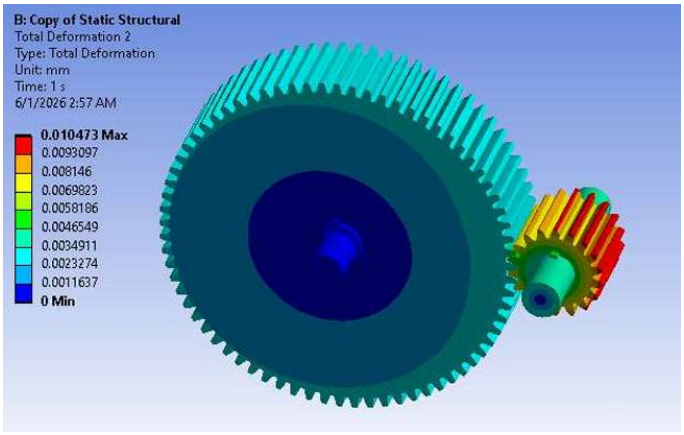


Fig 4: deformation of Gear and pinion.

Static analysis showed that, for both components, maximum deformation occurred at the tooth tip—where support is least—while minimum deformation occurred at the hub, consistent with

International Journal Of Innovative Trends In Multi Disciplinary Scientific Research
ISSN (Online): 3139 – 2687
(www.ijtmsr.co.in) Volume 9, Issue 6, 2026

cantilever-type bending of the tooth. Maximum equivalent stress was concentrated at the tooth–disk fillet region and at sharp corners, while the remainder of each body experienced comparatively low stress, indicating the design operates safely within allowable limits at all four speeds. Tables I–IV summarize the analytical (MATLAB) and numerical (ANSYS) stress and deformation results for the gear and pinion.

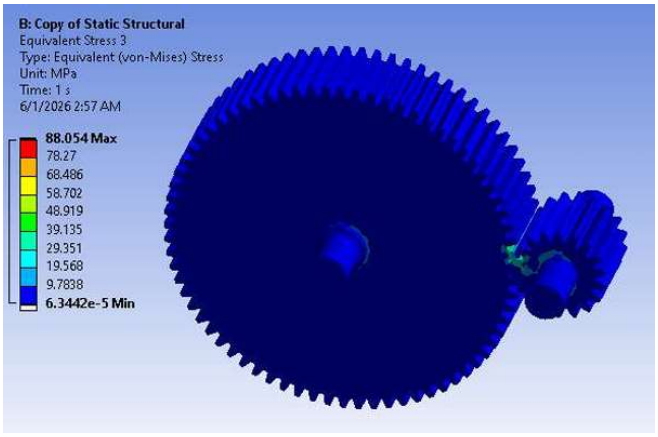


Fig 5: stresses on Gear and pinion.

TABLE I. STRESS RESULTS FOR GEAR

Torque (N·m)	Speed (RPM)	MATLAB (MPa)	ANSYS (MPa)
32156.51	3500	53.03	51.35
28136.94	4000	46.40	44.93
25010.62	4500	41.25	39.94
22509.55	5000	37.12	35.94

TABLE II. STRESS RESULTS FOR PINION

Torque (N·m)	Speed (RPM)	MATLAB (MPa)	ANSYS (MPa)
32156.51	3500	87.57	88.05
28136.94	4000	76.63	77.05
25010.62	4500	68.11	68.49
22509.55	5000	61.30	61.64

The stress in both components decreases monotonically with increasing speed because the transmitted torque falls proportionally (P = constant). The gear stress reduced by about 30% from the 3500 RPM to the 5000 RPM case, with MATLAB–ANSYS deviation of only 3.0–3.2%. The pinion, being smaller and having fewer teeth, consistently carries higher stress than the gear (up to 88 MPa versus 51 MPa at 3500 RPM), confirming it as the critical component in the pair; here the MATLAB–ANSYS deviation was even smaller, at 0.4–0.7%.

TABLE III. DEFORMATION RESULTS FOR GEAR AND PINION

Speed (RPM)	Gear MATLAB (mm)	Gear ANSYS (mm)	Pinion MATLAB (mm)	Pinion ANSYS (mm)
3500	0.00681	0.0053	0.00797	0.0079
4000	0.00596	0.0047	0.00698	0.0069
4500	0.00530	0.0042	0.00620	0.0062
5000	0.00477	0.0037	0.00558	0.0056

Deformation followed the same decreasing trend as stress with increasing speed, with maximum deformation not exceeding 0.0080 mm for either component across all load cases—well within acceptable design limits and indicative of high structural stiffness. Minor differences between MATLAB (beam-theory) and ANSYS (solid FE) deformation values are attributed to the simplified cantilever-beam idealization used in the analytical model versus the full 3-D solid representation used in FEA.

B. Modal Analysis Results

Linear modal analysis was performed to extract the first three natural frequencies and corresponding mode shapes of the gear and pinion. For both components, Mode 1 corresponded to tooth deformation extending along the axial length of the tooth with a twisting response, while Modes 2 and 3 corresponded to bending-dominated tooth response in the transverse direction. Table IV presents a comparison of the MATLAB (cantilever-beam) and ANSYS (solid FE) natural frequencies.

TABLE IV. NATURAL FREQUENCIES OF GEAR AND PINION

International Journal Of Innovative Trends In Multi Disciplinary Scientific Research
ISSN (Online): 3139 – 2687
(www.ijtmsr.co.in) Volume 9, Issue 6, 2026

Mode	Gear MATLAB (Hz)	Gear ANSYS (Hz)	Pinion MATLAB (Hz)	Pinion ANSYS (Hz)
1	79,547.97	116,800	92,186.32	125,270
2	498,554.6	503,100	577,763.5	572,790
3	1,396,108	1,381,000	1,617,917	1,600,300

For Mode 1, the deviation between MATLAB and ANSYS is comparatively large (35.9% for the pinion and 46.9% for the gear), attributable to the simplified single-tooth cantilever-beam assumption used analytically, which does not capture the coupling between tooth and disk stiffness present in the full solid model. However, for Modes 2 and 3—where bending response dominates and is better represented by beam theory—agreement improves markedly, to within 0.9–1.1% for both components. In all cases, the lowest natural frequency obtained (≈1.17–1.25 × 10^5 Hz) is several orders of magnitude higher than the maximum shaft rotational frequency corresponding to 5000 RPM (≈83.3 Hz) and its gear-mesh harmonics, indicating a negligible risk of resonance during normal operation of the transmission system.

V. CONCLUSION

Structural and modal analyses of an AISI 4340 spur gear and pinion assembly were carried out using CATIA for solid modeling, ANSYS Workbench for finite element analysis, and MATLAB for analytical validation, across four operating conditions between 3500 and 5000 RPM. Maximum stress in the gear decreased from 51.35 to 35.94 MPa and in the pinion from 88.05 to 61.64 MPa with increasing speed, while corresponding maximum deformations remained below 0.0080 mm throughout, confirming adequate strength and stiffness. MATLAB and ANSYS results showed excellent correlation for stress and deformation (within about 3%) and for the second and third natural frequency modes (within about 1%), validating the analytical model. All computed natural frequencies were found to lie well above the operating excitation frequencies, indicating that resonance is unlikely during service. It is concluded that the designed gear and pinion assembly is structurally safe, dynamically stable, and suitable for reliable operation in the intended power transmission application.

VI. FUTURE SCOPE

Future work may extend this study to alternative gear materials, including composites, titanium alloys and high-strength steels, to improve the strength-to-weight ratio and fatigue life. Optimized tooth profile modifications such as profile shifting and crowning could be investigated to reduce stress concentration. Dynamic and fatigue analyses under variable and real operating loads, incorporation of lubrication, wear and thermal effects into the numerical model, and the use of AI-based and topology optimization techniques for lightweight gear design are recommended as directions for further research.

VII. REFERENCES

[1] G. Tian, Z. Gao, P. Liu, and Y. Bian, "Dynamic modeling and stability analysis for a spur gear system considering gear backlash and bearing clearance," Machines, vol. 10, no. 6, p. 439, 2022.

[2] Y. Wang, P. Liu, and D. Dou, "Investigation of load capacity of high-contact-ratio internal spur gear drive with arc path of contact," Applied Sciences, vol. 12, no. 7, p. 3345, 2022.

[3] Z. Zhao et al., "An improved model for meshing characteristics analysis of spur gears considering fractal surface contact and friction," Mechanism and Machine Theory, vol. 158, p. 104219, 2021.

[4] T. G. Yılmaz, O. Doğan, and F. Karpat, "A numerical investigation on the hybrid spur gears: Stress and dynamic analysis," Proc. IMechE Part C, 2021.

[5] F. Zheng, J. Zhang, L. Yao, and R. Tan, "Investigation on the wear of spur gears generated by modified cutter," Friction, vol. 9, no. 2, pp. 288–300, 2021.

[6] S. M. Demet and A. S. Ersoyoğlu, "An analysis of the effect of pressure angle change on bending fatigue performance in asymmetrical spur gears," Proc. IMechE Part L, vol. 235, no. 9, pp. 2142–2150, 2021.

[7] J. Lin et al., "A quantitatively distributed wear-measurement method for spur gears during micro-pitting and pitting tests," Tribology International, vol. 157, p. 106839, 2021.

[8] Y. Zhang, C. Purssell, K. Mao, and S. Leigh, "A physical investigation of wear and thermal characteristics of 3D printed nylon spur gears," Tribology International, vol. 141, p. 105953, 2020.

[9] M. Pleguezuelos, M. B. Sánchez, and J. I. Pedrero, "Control of transmission error of high contact ratio spur gears with symmetric profile modifications," Mechanism and Machine Theory, vol. 149, p. 103839, 2020.

[10] F. Karpat, C. Yuce, and O. Doğan, "Experimental measurement and numerical validation of single tooth stiffness for involute spur gears," Measurement, vol. 150, p. 107043, 2020.

[11] K. Chen et al., "Calculation of mesh stiffness of spur gears considering complex foundation types and crack propagation paths," Mechanical Systems and Signal Processing, vol. 130, pp. 273–292, 2019.

[12] K. Feng et al., "Vibration-based updating of wear prediction for spur gears," Wear, vol. 426, pp. 1410–1415, 2019.

[13] O. Doğan and F. Karpat, "Crack detection for spur gears with asymmetric teeth based on the dynamic transmission error," Mechanism and Machine Theory, vol. 133, pp. 417–431, 2019.

[14] P. K. Singh and A. K. Singh, "An investigation on the thermal and wear behavior of polymer based spur gears," Tribology International, vol. 118, pp. 264–272, 2018.

[15] A. K. Singh, Siddhartha, and P. K. Singh, "Polymer spur gears behaviors under different loading conditions: A review," Proc. IMechE Part J, vol. 232, no. 2, pp. 210–228, 2018.

[16] A. Diez-Ibarbia et al., "Frictional power losses on spur gears with tip reliefs. The friction coefficient role," Mechanism and Machine Theory, vol. 121, pp. 15–27, 2018.

[17] S.-C. Hwang, J.-H. Lee, D.-H. Lee, S.-H. Han, and K.-H. Lee, "Contact stress analysis for a pair of mating gears," Mathematical and Computer Modelling, vol. 57, pp. 40–49, 2013.

[18] R. Prabhu Sekar and R. Sathishkumar, "Enhancement of wear resistance on normal contact ratio spur gear pairs through non-standard gears," Wear, 2017.

[19] I. S. Al-Tubi, H. Long, J. Zhang, and B. Shaw, "Experimental and analytical study of gear micropitting initiation and propagation under varying loading conditions," Wear, vol. 328–329, pp. 8–16, 2015.

[20] X. Dai, C. G. Cooley, and R. G. Parker, "Dynamic tooth root strains and experimental correlations in spur gear pairs," Mechanism and Machine Theory, vol. 101, pp. 60–74, 2016.

[21] S. Li, "Centrifugal load and its effects on bending strength and contact strength of a high speed thin-walled spur gear with offset web," Mechanism and Machine Theory, vol. 43, pp. 217–239, 2008.

[22] X. Liang, H. Zhang, L. Liu, and M. J. Zuo, "The influence of tooth pitting on the mesh stiffness of a pair of external spur gears," Mechanism and Machine Theory, vol. 106, pp. 1–15, 2016.

[23] T. Osman and Ph. Velex, "A model for the simulation of the interactions between dynamic tooth loads and contact fatigue in spur gears," Tribology International, vol. 46, pp. 84–96, 2012.

[24] S. S. Patil, S. Karuppanan, and I. Atanasovska, "Experimental measurement of strain and stress state at the contacting helical gear pairs," Measurement, vol. 82, pp. 313–322, 2016.

[25] A. Saxena, A. Parey, and M. Cooksey, "Effect of shaft misalignment and friction force on time varying mesh stiffness of spur gear pair," Engineering Failure Analysis, vol. 49, pp. 79–91, 2015.

[26] M. B. Sánchez, M. Pleguezuelos, and J. I. Pedrero, "Calculation of tooth bending strength and surface durability of internal spur gear drives," Mechanism and Machine Theory, vol. 95, pp. 102–113, 2016.

[27] J. Wu, Y. Yang, X. Yang, and J. Cheng, "Fault feature analysis of cracked gear based on LOD and analytical-FE method," Mechanical Systems and Signal Processing, vol. 98, pp. 951–967, 2018.

[28] P. B. Pawar and A. A. Utpat, "Analysis of composite material spur gear under static loading condition," Materials Today: Proceedings, vol. 2, pp. 2968–2974, 2015.

[29] M. Franulovic, K. Markovic, Z. Vrcan, and M. Soban, "Experimental and analytical investigation of the influence of pitch deviations on the loading capacity of HCR spur gears," Mechanism and Machine Theory, vol. 117, pp. 96–113, 2017.

[30] N. K. Raghuwanshi et al., "Effect of back-side contact on mesh stiffness of spur gear pair by finite element method," Procedia Engineering, vol. 173, pp. 1538–1543, 2017.

[31] P. Kumar, H. Hirani, and A. Agrawal, "Fatigue failure prediction in spur gear pair using AGMA approach," Materials Today: Proceedings, vol. 4, pp. 2470–2477, 2017.

[32] M. B. Sánchez, M. Pleguezuelos, and J. I. Pedrero, "Approximate equations for the meshing stiffness and the load sharing ratio of spur gears including hertzian effects," Mechanism and Machine Theory, vol. 109, pp. 231–249, 2017.

[33] P. Srinivasa Rao, N. Sriraj, and M. Farookh, "Contact stress analysis of spur gear for different materials using ANSYS and Hertz equation," International Journal of Modern Studies in Mechanical Engineering, vol. 1, no. 1, pp. 45–52, 2015.

[34] D. Potghan and S. Sharma, "Finite element analysis of spur gear used in lathe headstock," International Journal of Engineering Sciences & Research Technology, vol. 4, no. 7, 2015.

[35] A. M. Sankpal and M. M. Mirza, "Contact stress analysis of spur gear by photoelastic technique and finite element analysis," International Journal of Engineering Research, vol. 3, no. 2, 2014.

[36] T. Narawade, "Contact stress analysis and stress reduction of spur gear by using analytical and experimental method," International Engineering Research Journal, pp. 664–669.

[37] S. K. Khavdu and K. M. Vyas, "Comparative finite element analysis of metallic spur gear and hybrid spur gear," International Journal of Advanced Research in Engineering and Technology, vol. 6, no. 4, pp. 117–125, 2015.

[38] D. B. Vaitkar and P. N. Ulhe, "Experimental investigation and finite element analysis of contact stresses of spur gear used in automated lathe machine," International Journal of Innovative and Emerging Research in Engineering, vol. 4, no. 6, 2017.